

Senior Research Project

Wave Diffusion

Professor Bill Siegmann

Every applied mathematician knows that oscillations such as sound waves satisfy a wave equation, and that diffusive properties such as heat conduction satisfy a diffusion equation. Would you ever expect “waves” to “diffuse”? This type of behavior may happen if the medium in which the wave propagates is sufficiently complex. The complexity is modeled mathematically as a random medium, where properties like wave speed have small variations that are too complex to measure or to describe simply. We would like to develop and analyze a simple model of wave propagation in a random medium using a Galton Box (see Wikipedia, or the Boston Museum of Science). The objective is to include idealized features of upper ocean waveguides which result from ubiquitous phenomena known as internal waves (see Wikipedia, or the Boston Museum of Science), which have important influences on acoustic propagation.

Prerequisite: MATH-2400 Introduction to Differential Equations

Helpful: MATH-4700 Foundations of Applied Mathematics
MATH-4800 Numerical Computing

Collaborators: Scientists from the Woods Hole Oceanographic Institution

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Three Waves in Poro-Elastic Media

Professor Bill Siegmann

The propagation of acoustic waves in fluids, such as air or water, is part of everyday life. Acoustic waves also propagate in solids, which allow two types known as P (compressional) and S (shear) waves. How about in a poro-elastic medium, which is a solid matrix saturated by a fluid – like a sponge? Perhaps unsurprisingly, three types of acoustic waves propagate. The properties of these waves are determined by parameters that model the medium. For “typical values of interest” of the parameters, it happens that the third wave is significantly damped, in analogy with a mass-spring system which is overdamped instead of underdamped. We would like to analyze equations of a simplified poro-elastic model that exhibits this behavior. The objective is to understand acoustic propagation through mud-like sediments in the upper layer of the ocean bottom and in some locations on the earth’s surface.

Prerequisite: MATH-2400 Introduction to Differential Equations

Helpful: MATH-4700 Foundations of Applied Mathematics
MATH-4600 Advanced Calculus

Collaborators: Scientists from Boston University

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Waves in Rocks and Water

Professor Bill Siegmann

Consider a rock (a classical elastic solid) and water (a classical fluid), and the small motions that may occur in each. The motions are often very different, wouldn't you say? Now think about motions that are small-amplitude waves, either sound waves in water, or P and S waves in the rock. The mathematical descriptions of these waves have many interesting similarities. We would like to examine the wave equations for mathematical models of classical fluids and solids. The equations contain parameters that represent wave speeds that may vary with the spatial coordinates. We would like to see what happens for solids whose properties change to become more like fluids. The objective is to provide a basis for a unified treatment of acoustic propagation in fluids and materials that can be thought of as "soft rocks", which may be the sediments in the ocean bottom or in the upper layers of the earth.

Prerequisite: MATH-4700 Foundations of Applied Mathematics

Helpful: MATH-4600 Advanced Calculus

Collaborators: Scientists from the Naval Research Laboratory